



Test Two

This pleasant fifty-minute test covers the second and parts of the third chapter of “Proofs and Fundamentals” by Ethan Bloch. Each proof is worth 6 points.

At some point you may find the following result useful.

Theorem A: Let a and b be integers. If a prime p divides ab , then p divides a or p divides b .

Prove three of the following four theorems. Use a separate page for each proof. Other than Theorem A, you should only need definitions (even, odd, divides...), logic and basic algebra.

Theorem 1: The square root of 7 is irrational.

Theorem 2: Let a , b and c be integers. If $a \mid (b+c)$ and $a \mid c$, then $a \mid b$.

Theorem 3: Let n be an odd integer. Show there is an integer k such that $n^2 = 8k + 1$.

Theorem 4: If a and b are non-negative real numbers, then $\frac{a+b}{2} \geq \sqrt{ab}$

Use a separate page for each proof. Use only logic and the results above these on the attached sheet. Justify each step.

Theorem 5: Let A and B be sets. If $A = B$, then $B = A$.

Theorem 2.4 (i)

Theorem 3.3 (iv)