



Test Three

This fluffy and fun fifty-minute test covers parts of chapters three and four of Ethan Bloch's *Proofs and Fundamentals*. All parts of problems are five points unless otherwise stated.

1. Circle T for (always) true or F for (at least once) false. Let A, B and C be sets.

(1 point each)

T F $\{\emptyset\}$ is subset of every power set.

T F The set $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset, \{\emptyset\}\}\}$ has more than three elements.

T F The empty set is a proper subset of every set.

T F Every function is a set.

T F For all sets A and B, exactly one of $A = B$, $A \subseteq B$ and $B \subseteq A$ must be true.

T F $\wp(A \times B) = \wp(A) \times \wp(B)$

T F $A \times \emptyset = \emptyset$.

T F If $A \cup B \subseteq A \cap B$, then $A=B$.

T F If $f: \mathbb{R} \rightarrow \mathbb{R}$, by $f(x) = \lfloor x \rfloor$, then $f(f^{-1}(\{2,3\})) = [2,3)$.

T F If $A \subseteq C$ and $f: C \rightarrow C$, then $f(f^{-1}(A)) \subseteq A$.

2. Let $A = \{2,10,12\}$, $B=(2,5)$, $C=[4,6]$. Find the following.

(3 points each)

a. $\wp(A)$

b. $B - (C - A)$

c. $(A \cup B) \cap C$

Prove the following five theorems: (A, B, C and D are sets in all theorems.)

3. **Theorem.** If $A \subseteq B$, then $C - B \subseteq C - A$.

4. **Theorem.** If $A \subseteq C$, $B \subseteq C$ and $f: C \rightarrow D$, then $f(A) - f(B) \subseteq f(A - B)$

5. **Theorem.** If $A \subseteq D$, $B \subseteq D$ and $f: C \rightarrow D$, then $f^{-1}(A) - f^{-1}(B) \subseteq f^{-1}(A - B)$

6. **Theorem.** If $A \subseteq B$, then $\wp(A) \subseteq \wp(B)$

7. **Theorem.** $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$