Math  Test 4 ______________________

To best enjoy this succulent 6-page, 50-minute, 47-point test: relax, scan the whole test, and then write to be read. Use your time wisely.

1. Carefully define each of the following terms in the space provided. Let A and B be sets. (2 points each)

a) $\underline{f^{-1}(C)}$ for a function $f{:}A\to B$ and a set $C \subseteq B$.

b) Left inverse of a function $f{:}A\to B$

c) A relation R from A to B

d) The relation R on A is symmetric

2. For each of the following, circle T for always true or F for at least once false. (1 point each)

T F If a function has two different left inverses, then it has a right inverse.

T F A function has an inverse iff it is a one-to-one and an injection.

T F Every identity function has an inverse.

T F $1234^{5678} \cdot 90 \equiv 0 \pmod{12}$

T F If $f{:}A\to B$ is a function and $S\subseteq A$. If f has a right inverse, so does $f|_S$.

T F If the functions $f{:}A\to B$ and $g{:}B\to A$ have inverses, then $f \circ g$ is defined.

T F If n is odd, then $n^2 \equiv 1 \pmod{8}$.

T F If $f{:}A\to A$ and the range of f equals the domain of f, then f is surjective (onto).

3. For each of the following relations, circle R for reflexive, S for symmetric, T for transitive and E for equivalence relation. (Obviously more than one may be circled!) (2 points each)

R S T E { (1,1), (2,3), (3,2), (1,3), (3,1) } on {1,2,3}

R S T E Let P be the set of all people. Let x R y iff x and y have the same color eyes or the same color hair (for all x, y in P).

R S T E $A \times A$ on any non-empty set A.

4. Find [3], the relations class of 3, for the following relations. (2 points each)

a. For all x, y in the reals, $x \sim y$ iff $x = \sin(y)$.

b. For all x, y in the integers, $x \sim y$ iff x divides y.

5. Give an example of a function from the integers to the integers that is surjective but not bijective. (1 point)

6. Prove the following theorems. Turn them in on separate pieces of paper. (5 points each)

Theorem H. Let $f:A \to B$ be a function and let $C \subseteq A$, $S \subseteq B$ be sets. If $f(C) \subseteq S$, then $C \subseteq f^{-1}(S)$.

Theorem E. Let $f:A \to B$ and $g:B \to C$ be functions between sets. If f and g are both have inverses, then $f^{-1} \circ g^{-1}$ is the inverse of $g \circ f$.

Theorem L. Let $f:A \to B$ and $g:B \to C$ be functions between sets. If $g \circ f$ is surjective, then g is surjective.

Theorem P: Let R be an equivalence relation on the set A and $x, y \in A$. If $[x] \cap [y] \neq \emptyset$, then $[x] = [y]$.