Math 314 Test Five ______________________________

Puny-pink-pixilated-ponies! That's a lot of definitions batman!

This is sure an easy 6-page, 50-minute, 43-point test: relax, scan the whole test, and then write to be read. Use your time wisely.

1. Carefully define each of the following terms in the space provided. Let A and B be sets. (2 points each)

a) The function $f:A\to B$ is a surjection.

b) The function $f:A\to B$ is an injection if

c) The set A is finite if

d) The set A is countably infinite if

e) The set A is uncountable if

f) Sets A and B have the same cardinality if

2. For each of the following, circle T for always true or F for at least once false. (1 point each)

T F A function is a bijection if and only if it has an inverse.

T F The set of reals is countably infinite.

T F The set of rationals is uncountable.

T F Two sets have the same cardinality iff they have the same number of elements.

T F A set is infinite iff it is equivalent to a proper subset of itself.

T F I (the person taking this test) marked this statement false.

T F A set can be both uncountable and infinite.

3. Prove the following theorems. Turn them in on separate pieces of paper. (5 points each)

Theorem A: For all positive integers n, $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \ldots + n \cdot n! = (n+1)! - 1$.

Theorem B. For all integers $n \geq 0$, $\left(2^{2^0}+1\right)\left(2^{2^1}+1\right)\left(2^{2^2}+1\right)\ldots\left(2^{2^n}+1\right) = 2^{2^{n+1}}-1$.

Theorem C. $2^{n-1} \leq n!$ for all positive integers n.

Theorem D: If set A has the same cardinality as set B, and set B has the same cardinality as set C, then set A has the same cardinality as set C.

Theorem E: The interval (0,1) has the same cardinality as the interval (241,314).